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Research Article

Well-Posedness and Fractals via Fixed Point Theory

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The purpose of this paper is to present existence, uniqueness, and data dependence results for the strict fixed points of a multivalued operator of Reich type, as well as, some sufficient conditions for the well-posedness of a fixed point problem for the multivalued operator.

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1. Introduction

Let (X, d) be a metric space. We will use the following symbols (see also [1]):

$$P(X) = \{Y \subset X \mid Y \neq \emptyset\};$$

$$P_b(X) = \{Y \in P(X) \mid Y \text{ is bounded}\};$$

$$P_{cl}(X) = \{Y \in P(X) \mid Y \text{ is closed}\};$$

$$P_{cp}(X) = \{Y \in P(X) \mid Y \text{ is compact}\}.$$

If $T : X \rightarrow P(X)$ is a multivalued operator, then for $Y \in P(X)$, $T(Y) = \bigcup_{x \in Y} T(x)$ we will denote the image of the set Y through T .

Throughout the paper $F_T := \{x \in X \mid x \in T(x)\}$ (resp., $(SF)_T := \{x \in X \mid \{x\} = T(x)\}$) denotes the fixed point set (resp., the strict fixed point set) of the multivalued operator T .

We introduce the following generalized functionals.

The δ generalized functional

$$\begin{aligned} \delta_d : P(X) \times P(X) &\longrightarrow \mathbb{R}_+ \cup \{+\infty\}, \\ \delta_d(A, B) &= \sup \{d(a, b) \mid a \in A, b \in B\}. \end{aligned} \tag{1.1}$$

The gap functional

$$\begin{aligned} D_d : P(X) \times P(X) &\longrightarrow \mathbb{R}_+ \cup \{+\infty\}, \\ D_d(A, B) &= \inf \{d(a, b) \mid a \in A, b \in B\}. \end{aligned} \quad (1.2)$$

The excess generalized functional

$$\begin{aligned} \rho_d : P(X) \times P(X) &\longrightarrow \mathbb{R}_+ \cup \{+\infty\}, \\ \rho_d(A, B) &= \sup \{D_d(a, B) \mid a \in A\}. \end{aligned} \quad (1.3)$$

The Pompeiu-Hausdorff generalized functional

$$\begin{aligned} H_d : P(X) \times P(X) &\longrightarrow \mathbb{R}_+ \cup \{+\infty\}, \\ H_d(A, B) &= \max \{\rho_d(A, B), \rho_d(B, A)\}. \end{aligned} \quad (1.4)$$

The first purpose of this paper is to present existence, uniqueness, and data dependence results for the strict fixed point of a multivalued operator of Reich type. Since, in our approach, the strict fixed point is constructed by iterations, this generates the possibility to give some sufficient conditions for the well-posedness of a fixed point problem for the multivalued operator mentioned below.

Definition 1.1. Let (X, d) be a metric space and $T : X \rightarrow P_{cl}(X)$. Then T is called a multivalued δ -contraction of Reich type, if there exist $a, b, c \in \mathbb{R}_+$ with $a + b + c < 1$ such that

$$\delta(T(x), T(y)) \leq ad(x, y) + b\delta(x, T(x)) + c\delta(y, T(y)), \quad (1.5)$$

for all $x, y \in X$.

The notion of well-posed fixed point problem for single valued and multivalued operator was defined and studied by F.S. De Blasi and J. Myjak, S. Reich and A.J. Zaslavski, Rus and Petruşel [2], Petruşel et al. [3].

Definition 1.2 (see Petruşel and Rus [2] and [3]). (A) Let (X, d) be a metric space, $Y \in P(X)$ and $T : Y \rightarrow P_{cl}(X)$ be a multivalued operator.

Then the fixed point problem is well posed for T with respect to D_d if

- (a₁) $F_T = \{x^*\}$ (i.e., $x^* \in T(x^*)$);
- (b₁) If $x_n \in Y$, $n \in \mathbb{N}$ and $D_d(x_n, T(x_n)) \rightarrow 0$ as $n \rightarrow \infty$ then $x_n \rightarrow x^*$ as $n \rightarrow \infty$.

(B) Let (X, d) be a metric space, $Y \in P(X)$ and $T : Y \rightarrow P_{cl}(X)$ be a multivalued operator.

Then the fixed problem is well posed for T with respect to H_d if

- (a₂) $(SF)_T = \{x^*\}$ (i.e., $\{x^*\} = T(x^*)$);
- (b₂) If $x_n \in Y$, $n \in \mathbb{N}$ and $H_d(T(x_n)) \rightarrow 0$ as $n \rightarrow \infty$ then $x_n \rightarrow x^*$ as $n \rightarrow \infty$.

The second aim is to study the existence of an attractor (i.e., the fixed point of the multifractal operator, see [4–7]) for an iterated multifunction system consisting of nonself multivalued operators.

2. Main results

We will give first another proof (a constructive one) of a result given by Reich [8] in 1972. For some similar results, see [9, 10]. In our proof, the strict fixed point will be obtained by iterations.

Theorem 2.1 (Reich's theorem). *Let (X, d) be a complete metric space and let $T : X \rightarrow P_b(X)$ be a multivalued operator, for which there exist $a, b, c \in \mathbb{R}_+$ with $a + b + c < 1$ such that*

$$\delta(T(x), T(y)) \leq ad(x, y) + b\delta(x, T(x)) + c\delta(y, T(y)), \quad \forall x, y \in X. \quad (2.1)$$

Then T has a unique strict fixed point in X , that is, $(SF)_T = \{x^\}$.*

Proof. Let $q > 1$ and $x_0 \in X$ be arbitrarily chosen. Then there exists $x_1 \in T(x_0)$ such that

$$\delta(x_0, T(x_0)) \leq qd(x_0, x_1). \quad (2.2)$$

We have

$$\begin{aligned} \delta(x_1, T(x_1)) &\leq \delta(T(x_0), T(x_1)) \\ &\leq ad(x_0, x_1) + b\delta(x_0, T(x_0)) + c\delta(x_1, T(x_1)) \\ &\leq (a + bq)d(x_0, x_1) + c\delta(x_1, T(x_1)). \end{aligned} \quad (2.3)$$

It follows that

$$\delta(x_1, T(x_1)) \leq \frac{a + bq}{1 - c} d(x_0, x_1). \quad (2.4)$$

For $x_1 \in T(x_0)$, there exists $x_2 \in T(x_1)$ such that

$$\delta(x_1, T(x_1)) \leq qd(x_1, x_2). \quad (2.5)$$

Then

$$\begin{aligned} \delta(x_2, T(x_2)) &\leq \delta(T(x_1), T(x_2)) \\ &\leq ad(x_1, x_2) + b\delta(x_1, T(x_1)) + c\delta(x_2, T(x_2)) \\ &\leq (a + bq)d(x_1, x_2) + c\delta(x_2, T(x_2)). \end{aligned} \quad (2.6)$$

It follows that

$$\begin{aligned}
 \delta(x_2, T(x_2)) &\leq \frac{a+bq}{1-c} d(x_1, x_2) \\
 &\leq \frac{a+bq}{1-c} \delta(x_1, T(x_1)) \\
 &\leq \left(\frac{a+bq}{1-c} \right)^2 d(x_0, x_1).
 \end{aligned} \tag{2.7}$$

Inductively, we can construct a sequence $(x_n)_{n \in \mathbb{N}}$ having the properties

- (1) $(\alpha)x_n \in T(x_{n-1}), n \in \mathbb{N}^*$;
- (2) $(\beta)d(x_n, x_{n+1}) \leq \delta(x_n, T(x_n)) \leq ((a+bq)/(1-c))^n d(x_0, x_1)$.

We will prove now that the sequence $(x_n)_{n \in \mathbb{N}}$ is Cauchy.

We successively have

$$\begin{aligned}
 d(x_n, x_{n+p}) &\leq d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \cdots + d(x_{n+p-1}, x_{n+p}) \\
 &\leq \left[\left(\frac{a+bq}{1-c} \right)^n + \left(\frac{a+bq}{1-c} \right)^{n+1} + \cdots + \left(\frac{a+bq}{1-c} \right)^{n+p-1} \right] d(x_0, x_1).
 \end{aligned} \tag{2.8}$$

Let us denote $\alpha := (a+bq)/(1-c)$. Then

$$d(x_n, x_{n+p}) \leq \alpha^n (1 + \alpha + \cdots + \alpha^{p-1}) d(x_0, x_1) = \alpha^n \frac{\alpha^p - 1}{\alpha - 1} d(x_0, x_1). \tag{2.9}$$

If we chose $q < (1-a-c)/b$, then $\alpha < 1$.

Letting $n \rightarrow \infty$, since $\alpha^n \rightarrow 0$, it follows that

$$d(x_n, x_{n+p}) \longrightarrow 0 \quad \text{as } n \longrightarrow \infty. \tag{2.10}$$

Hence $(x_n)_{n \in \mathbb{N}}$ is Cauchy.

By the completeness of the space (X, d) , we get that there exists $x^* \in X$ such that $x_n \rightarrow x^*$ as $n \rightarrow \infty$.

Next, we will prove that $x^* \in (SF)_T$.

We have

$$\begin{aligned}
 \delta(x^*, T(x^*)) &\leq d(x^*, x_n) + \delta(x_n, T(x_n)) + \delta(T(x_n), T(x^*)) \\
 &\leq d(x^*, x_n) + \delta(x_n, T(x_n)) + ad(x_n, x^*) + b\delta(x_n, T(x_n)) + c\delta(x^*, T(x^*)).
 \end{aligned} \tag{2.11}$$

Then

$$\delta(x^*, T(x^*)) \leq \frac{1+a}{1-c}d(x^*, x_n) + \frac{1+b}{1-c}\delta(x_n, T(x_n)) \quad (2.12)$$

because $\delta(x_n, T(x_n)) \leq \alpha^n d(x_0, x_1) \Rightarrow \delta(x^*, T(x^*)) = 0 \Rightarrow T(x^*) = \{x^*\}$ (i.e., $x^* \in (SF)_T$).

For the last part of our proof, we will show the uniqueness of the strict fixed point.

Suppose that there exist $x^*, y^* \in (SF)_T$. Then

$$d(x^*, y^*) = \delta(T(x^*), T(y^*)) \leq ad(x^*, y^*) + b\delta(x^*, T(x^*)) + c\delta(y^*, T(y^*)). \quad (2.13)$$

If x^* and y^* are distinct points, then we get that $a \geq 1$, which contradicts our hypothesis. Thus $x^* = y^*$. The proof is complete. \square

Regarding the well-posedness of a fixed point problem, we have the following result.

Theorem 2.2. *Let (X, d) be a complete metric space and let $T : X \rightarrow P_b(X)$ be a multivalued operator. Suppose there exist $a, b, c \in \mathbb{R}_+$ with $a + b + c < 1$ such that*

$$\delta(T(x), T(y)) \leq ad(x, y) + b\delta(x, T(x)) + c\delta(y, T(y)), \quad \forall x, y \in X. \quad (2.14)$$

Then the fixed point problem is well posed for T with respect to H_d .

Proof. By Reich's theorem, we get that $(SF)_T = \{x^*\}$.

Let $x_n \in X, n \in \mathbb{N}$ such that $H_d(x_n, T(x_n)) \rightarrow 0$ as $n \rightarrow \infty$. Then

$$H_d(x_n, T(x_n)) = \delta_d(x_n, T(x_n)). \quad (2.15)$$

We have to show that $x_n \rightarrow x^*$ as $n \rightarrow \infty$. We successively have

$$\begin{aligned} d(x_n, x^*) &\leq \delta_d(x_n, T(x_n)) + \delta_d(T(x_n), T(x^*)) \\ &\leq \delta_d(x_n, T(x_n)) + ad(x_n, x^*) + b\delta_d(x_n, T(x_n)) + c\delta_d(x^*, T(x^*)) \\ &= (1+b)\delta_d(x_n, T(x_n)) + ad(x_n, x^*). \end{aligned} \quad (2.16)$$

It follows that

$$d(x_n, x^*) \leq \frac{1+b}{1-a}\delta_d(x_n, T(x_n)) = \frac{1+b}{1-a}H_d(x_n, T(x_n)) \rightarrow 0, \quad n \rightarrow \infty. \quad (2.17)$$

Hence

$$x_n \rightarrow x^*, \quad n \rightarrow \infty. \quad (2.18)$$

\square

With respect to the same multivalued operators, a data dependence result can also be established as follows.

Theorem 2.3. Let (X, d) be a complete metric space and let $T_1, T_2 : X \rightarrow P_b(X)$ be two multivalued operators. Suppose that

(i) there exist $a, b, c \in \mathbb{R}_+$ with $a + b + c < 1$ such that

$$\delta(T_1(x), T_1(y)) \leq ad(x, y) + b\delta(x, T_1(x)) + c\delta(y, T_1(y)), \quad \forall x, y \in X \quad (2.19)$$

(denote the unique strict fixed point of T_1 by x_1^*);

(ii) $(SF)_{T_2} \neq \emptyset$;

(iii) there exists $\eta > 0$ such that $\delta(T_1(x), T_2(x)) \leq \eta$, for all $x \in X$.

Then

$$\delta(x_1^*, (SF)_{T_2}) \leq \frac{(1+c)\eta}{1-a}. \quad (2.20)$$

Proof. Let $x_2^* \in (SF)_{T_2}$. Then $\delta(x_2^*, T_2(x_2^*)) = 0$.

We have

$$\begin{aligned} d(x_1^*, x_2^*) &= \delta(T_1(x_1^*), T_2(x_2^*)) \\ &\leq \delta(T_1(x_1^*), T_1(x_2^*)) + \delta(T_1(x_2^*), T_2(x_2^*)) \\ &\leq ad(x_1^*, x_2^*) + b\delta(x_1^*, T_1(x_1^*)) + c\delta(x_2^*, T_1(x_2^*)) + \eta \\ &= ad(x_1^*, x_2^*) + c\delta(T_2(x_2^*), T_1(x_2^*)) + \eta \leq ad(x_1^*, x_2^*) + (1+c)\eta. \end{aligned} \quad (2.21)$$

It follows that

$$d(x_1^*, x_2^*) \leq \frac{1+c}{1-a}\eta. \quad (2.22)$$

By taking $\sup_{x_2^* \in (SF)_{T_2}}$, it follows that

$$\delta(x_1^*, (SF)_{T_2}) \leq \frac{1+c}{1-a}\eta. \quad (2.23)$$

□

Let (X, d) be a complete metric space and let $F_1, \dots, F_m : X \rightarrow P(X)$ be a finite family of multivalued operators.

The system $F = (F_1, \dots, F_m)$ is said to be an iterated multifunction system.

The operator

$$\tilde{T}_F : P(X) \longrightarrow P(X), \quad \tilde{T}_F(Y) = \bigcup_{i=1}^m F_i(Y), \quad Y \in P(X) \quad (2.24)$$

is called the multifractal operator generated by the iterated multifunction system $F = (F_1, \dots, F_m)$.

Remark 2.4. (i) If $F_i : X \rightarrow P_{cp}(X)$ are multivalued α_i -contractions for each $i \in \{1, 2, \dots, m\}$, then the multifractal operator \tilde{T}_F is an α -contraction too, where $\alpha := \max\{\alpha_i \mid i \in \{1, \dots, m\}\}$ (Nadler Jr. [7]).

(ii) If $F_i : X \rightarrow P_{cp}(X)$ are multivalued φ_i -contractions (see [4]) for each $i \in \{1, 2, \dots, m\}$, then the multifractal operator \tilde{T}_F is an φ -contraction too, see Andres and Fišer [4] for the definitions and the result.

(iii) If $F = (F_1, \dots, F_m)$ is an iterated multifunction system, such that $F_i : X \rightarrow P_{cp}(X)$ is upper semicontinuous for each $i \in \{1, \dots, m\}$, then the multifractal operator

$$\tilde{T}_F : P_{cp}(X) \longrightarrow P_{cp}(X), \quad \tilde{T}_F(Y) = \bigcup_{i=1}^m F_i(Y) \quad (2.25)$$

is well defined. A fixed point $Y^* \in P_{cp}(X)$ of \tilde{T}_F is called an attractor of the iterated multifunction system F .

The following result is well known, see, for example, Granas and Dugundji [11].

Lemma 2.5. *Let (X, d) be a complete metric space, $x_0 \in X$, $r > 0$ and*

$$B := \tilde{B}(x_0, r) = \{x \in X \mid d(x, x_0) \leq r\}. \quad (2.26)$$

Let $f : B \rightarrow X$ be an α -contraction.

If $d(x_0, f(x_0)) \leq (1 - \alpha)r$, then f has a unique fixed point in B .

Our next result concerns with the existence of an attractor for an iterated multifunction system.

Theorem 2.6. *Let (X, d) be a complete metric space, $x_0 \in X$ and $r > 0$. Let $F_i : \tilde{B}(x_0, r) \rightarrow P_{cp}(X)$, $i \in \{1, \dots, m\}$ a finite family of multivalued operators.*

Suppose that

(i) F_i is an α_i -contraction, for each $i \in \{1, \dots, m\}$;

(ii) $\delta(x_0, F_i(x_0)) \leq (1 - \max\{\alpha_i \mid i \in \{1, \dots, m\}\})r$, for all $i \in \{1, \dots, m\}$.

Then there exists $Y^ \in \tilde{B}(\{x_0\}, r) \subset P_{cp}(X)$ a unique attractor of the iterated multifunction system $F = (F_1, \dots, F_m)$.*

Proof. Since $F_i : \tilde{B}(x_0, r) \rightarrow P_{cp}(X)$ is an α_i -contraction, for each $i \in \{1, \dots, m\}$ it follows that F_i is upper semicontinuous, for each $i \in \{1, \dots, m\}$. By Remark 2.4(iii), we get that the operator $\tilde{T}_F : \tilde{B}(\{x_0\}, r) \subset P_{cp}(X) \rightarrow P_{cp}(X)$, $\tilde{T}_F(Y) = \bigcup_{i=1}^m F_i(Y)$, $Y \in \tilde{B}(\{x_0\}, r)$ is well defined.

Any fixed point $Y^* \in \tilde{B}(\{x_0\}, r) \subset P_{cp}(X)$ of \tilde{T}_F is an attractor of the iterated multifunction system $F = (F_1, \dots, F_m)$.

Notice first that, if $Y \in \tilde{B}(\{x_0\}, r) \subset (P_{cp}(X), H)$, then $H(\{x_0\}, Y) \leq r$, which implies that $d(x_0, y) \leq r$, for all $y \in Y$. Thus $y \in \tilde{B}(x_0, r)$, for all $y \in Y$.

We will show that \tilde{T}_F satisfies the following two conditions:

(i) \tilde{T}_F is an α -contraction, with $\alpha := \max\{\alpha_i \mid i \in \{1, \dots, m\}\}$, that is,

$$H(\tilde{T}_F(Y_1), \tilde{T}_F(Y_2)) \leq \alpha H(Y_1, Y_2), \quad \forall Y_1, Y_2 \in \tilde{B}(\{x_0\}, r) \subset P_{cp}(X); \quad (2.27)$$

(ii) $H(\{x_0\}, \tilde{T}_F(\{x_0\})) \leq (1 - \alpha)r$.

Indeed, we have

(i) Let $Y_1, Y_2 \in \tilde{B}(\{x_0\}, r) \subset P_{cp}(X)$ și $u \in \tilde{T}_F(Y_1)$. By the definition of \tilde{T}_F , it follows that there exists $j \in \{1, \dots, m\}$ and there exists $y_1 \in Y_1$ such that $u \in F_j(y_1)$. Since $Y_1, Y_2 \in P_{cp}(X)$, there exists $y_2 \in Y_2$ such that $d(y_1, y_2) \leq H(Y_1, Y_2)$.

Since, for arbitrary $\varepsilon > 0$ and each $A, B \in P_{cp}(X)$ with $H(A, B) \leq \varepsilon$, we have that for all $a \in A$ there exists $b \in B$ such that $d(a, b) \leq \varepsilon$, by the following relations

$$H(F_j(y_1), F_j(y_2)) \leq \alpha_j d(y_1, y_2) \leq \alpha_j H(Y_1, Y_2), \quad (2.28)$$

we obtain that for $u \in F_j(y_1) \subset \tilde{T}_F(Y_1)$, there exists $v \in F_j(y_2) \subset \tilde{T}_F(Y_2)$ such that $d(u, v) \leq \alpha_j H(Y_1, Y_2) \leq \alpha H(Y_1, Y_2)$.

By the above relation and by the similar one (where the roles of $\tilde{T}_F(Y_1)$ and $\tilde{T}_F(Y_2)$ are reversed), the first conclusion follows.

(ii) We have to show that

$$\delta(\{x_0\}, \tilde{T}_F(\{x_0\})) \leq (1 - \alpha)r \quad (2.29)$$

or equivalently for all $u \in \tilde{T}_F(\{x_0\})$, we have $d(x_0, u) \leq (1 - \alpha)r$. Since $u \in \tilde{T}_F(\{x_0\})$ it follows that there exists $j \in \{1, \dots, m\}$ such that $u \in F_j(x_0)$. Then

$$d(x_0, u) \leq \delta(x_0, F_j(x_0)) \leq (1 - \alpha)r. \quad (2.30)$$

By Lemma 2.5, applied to \tilde{T}_F , we get that there exists $Y^* \in \tilde{B}(\{x_0\}, r) \subset P_{cp}(X)$ a unique fixed point for \tilde{T}_F , that is, a unique attractor of the iterated multifunction system $F = (F_1, \dots, F_m)$. The proof is complete. \square

Remark 2.7. An interesting extension of the above results could be the case of a set endowed with two metrics, see [12] for other details.

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